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ESTIMATION OF ADULT MORTALITY FROM PATERNAL ORPHANHOOD: A REASSESSMENT AND A NEW APPROACH

*Ian M. Timæus**

SUMMARY

This article proposes a new procedure for estimating men's mortality from paternal orphanhood which generally yields more accurate results than the existing approach. A procedure for estimating mortality from maternal orphanhood data based on consistent assumptions is also presented. The theory underlying these methods is outlined, focusing on aspects of it that have not been explained fully in the existing literature and that influence the specification and robustness of the models used for estimation. The article also points out an error made in the tabulation of the weighting factors used until now to estimate mortality from paternal orphanhood. Investigations using simulated data are presented which support the theoretical arguments that suggest that the paternal orphanhood method is more robust than has often been assumed and which confirm that the new approach usually produces more accurate estimates than the weighting factors.

The development of reliable and affordable methods for measuring adult mortality in countries that lack adequate vital statistics systems has proved a major challenge. While considerable ingenuity has been deployed to good effect to devise ways of rendering incomplete reports of recent deaths usable, such methods can be applied only when the majority of events are reported (Brass, 1975; Preston and others, 1980; Preston, 1984; Timæus and Graham, 1989). Thus, indirect techniques remain important sources of mortality estimates. This article proposes an improved procedure for making such estimates from data on the survival of fathers.

*Lecturer in Demography, Centre for Population Studies, London School of Hygiene and Tropical Medicine, London, United Kingdom.

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The first simple, robust method for estimating mortality from orphanhood was proposed by Brass and Hill (1973). It relates the proportions of respondents with living mothers or fathers in two adjoining age groups to measures of life-table survivorship by means of a system of weighting factors whose values depend on the mean age of child-bearing. Subsequent to the derivation of these weighting factors, several regression-based approaches to the estimation of women's mortality from data on maternal orphanhood have been proposed (Hill and Trussell, 1977; Palloni and Heligman, 1985; United Nations, 1983). Equivalent methods for the estimation of men's mortality from paternal orphanhood have not been developed because of the lack of a satisfactory, flexible model of male fertility and persistent scepticism about the robustness of the method (Brass, 1975; Hill and Trussell, 1977; Hill, 1984; Palloni and Heligman, 1985).

Despite the concern about the sensitivity of the paternal orphanhood method, little effort has been devoted to its evaluation. Research into the maternal orphanhood method has suggested that the weighting factors perform well for women's mortality at young ages but that a regression-based estimation procedure produces better results from data on older respondents (United Nations, 1983). Why this is so and whether it would also be true for paternal orphanhood remain unclear. Moreover, this analysis gives little clue as to which of the assumptions made in the derivation of the estimation procedures are crucial and whether there are particular conditions in which the accuracy of the estimation procedures is unacceptably low.

It is these issues, rather than reporting errors (Blacker and Mukiza-Gapere, 1988; Palloni, Massagli and Marcotte, 1984; Timæus, 1990 and 1991), that are considered here. The article outlines several theoretical considerations that affect the specification and robustness of the models used to estimate mortality from orphanhood. Using simulated data, it assesses the impact of breaches in the assumptions made by the estimation procedures on the results. On the basis of the preceding discussion, it proposes an improved method for analysis of paternal orphanhood data, which combines some of the advantages of both the weighting factors and the existing regression methods for analysing data on maternal orphanhood. A procedure for estimating women's mortality based on consistent assumptions is also presented. Finally, simulated data are used to assess the performance of the new paternal orphanhood method. To provide the basis for the discussion that follows, the rationale of Brass and Hill's approach is outlined briefly in the section below. The relationship between life-table survivorship and the survival of mothers is considered first.

Methodological background

If we consider respondents born a years before a demographic enquiry to women aged y and assume a stable age structure and that the mortality of orphans and children with living parents is the same (Blacker, 1984; Blacker and Mukiza-Gapere, 1988; Palloni, Massagli and Marcotte, 1984; Timæus, 1990), the proportion of respondents in a five-year age group with living mothers is (Brass and Hill, 1973):

$$ {}_5S_x = \frac{\int_x^{x+5} e^{-ra} l(a) \int_s^w e^{-ry} l(y+a) f(y)\, dy\, da}{\int_x^{x+5} e^{-ra} l(a) \int_s^w e^{-ry} l(y) f(y)\, dy\, da} \qquad (1) $$

where integration by y is over all ages at child-bearing, s to w; r is the rate of natural increase; and $f(y)$ is the fertility rate at age y.

There is no straightforward way of integrating either the numerator or the denominator of equation (1); thus it has to be evaluated numerically. Brass and Hill (1973) evaluate $e^{-ry} l(y+a) f(y)$ for five-year age groups and add the values together to approximate the integral. They take r, the rate of growth, as 2 per cent; the $l(a)$ values from Brass's (1971) General Standard; and represent fertility by a cubic function of the form $f(y) = (y - s)(s + 33 - y)^2$, where s is the age at which child-bearing starts.

Every value of ${}_5S_x$, for an age group with mid-point N, is equal to a survivorship ratio $l(B + N)/l(B)$ in the standard life-table used to generate the estimate of ${}_5S_x$. The value of B is close to $\bar{M}$, the unstandardized mean age at child-bearing, but is dependent on N. Instead of working with B and N, however, it is more convenient to adjust the proportions slightly and make estimates for b and n, where b is a fixed round numbered age near $\bar{M}$ and n is a multiple of five years. For the estimation of mortality from maternal orphanhood, Brass and Hill set b to 25 years and calculate the life-table survivorship ratios as weighted averages of the proportions of respondents with living mothers in the two age groups surrounding age n:

$$ {}_np_{25} = W_n \cdot {}_5S_{n-5} + (1 - W_n)\,{}_5S_n $$

where ${}_np_{25}$ is equivalent to $l(25 + n)/l(25)$. They present a table of these weights, W_n, for mean ages at child-bearing from 22 to 30 years.

Estimation of adult men's mortality from the proportion of respondents with living fathers is based on the same principles. It is possible for fathers to die between the time when a child is conceived and when it is born. This affects the relationships in equation (1) in several ways.

First, the age of the respondents becomes 0.75 years less than a, the duration of exposure of the fathers. Brass and Hill allow for this when they calculate the proportions of fathers surviving for five-year age groups of respondents from point estimates for a years of exposure.

Secondly, for paternal orphanhood the values of ${}_5S_x$ for an age group with a mid-point N are equivalent to a survivorship ratio $l(B + N + 0.75)/l(B)$ where B is, in this case, close to the mean age of fathers at the conception of their children. As for maternal orphanhood, it is convenient to calculate weighting factors for a fixed round age b and ages n that are multiples of 5 years. However, both because fathers' survival is being measured over an interval nine months longer than the respondents' ages and because of the high mortality of the men brought in by the long tail of the male fertility distribution, the amount of extrapolation from the observed proportions is reduced if survivorship is measured to the end of the upper age group. Therefore, Brass and Hill estimate life-table survivorship from data on the survival of fathers using:

$$l(b + n + 2.5)/l(b) = W_n \cdot {}_5S_{n-5} + (1 - W_n){}_5S_n$$

They present two tables of weighting factors with age b set to 32.5 years and 37.5 years, depending on the mean age of fathers at the birth of their children.

Thirdly, equation (1) only holds for men if the $f(y)$ distribution is treated as the distribution of fathers' ages at the conception of their live-born children, rather than as a fertility distribution. As a result, the mean age, $\bar{M}$, calculated from this distribution and the age structure in equation (1), represents the mean age of fathers at the conception of their live-born children. It is approximately three quarters of a year less than the mean age of child-bearing.

By recalculating the weighting factors using the original methodology, one can establish that no allowance was made for this difference in the interpretation of $\bar{M}$ in the tables of weighting factors for paternal orphanhood presented by Brass and Hill. Thus, each of the columns of weighting factors tabulated for integer values of the mean age of child-bearing in fact applies to a mean age of child-bearing 0.75 years greater.[1] For example, the weights tabulated for $\bar{M} = 33$ years are the correct weights for a mean age of child-bearing of 33.75 years. Almost all estimates of male survivorship in adulthood made from orphanhood data during the past 20 years have been calculated using these weighting factors. They are all biased systematically upward. Fortunately, the size of the bias is reasonably small, being equivalent, on average, to the overestimation of life expectancy at age 15 or 20 by about half a year.

ROBUSTNESS OF THE ESTIMATION PROCEDURES

Certain aspects of the rationale of the orphanhood method are not discussed in detail by either Brass and Hill or those who have contributed subsequently to this method's development. First, Brass and Hill do not explain why they adopt a series of weighting factors for estimating adult mortality from orphanhood, rather than the system of multiplying factors that Brass had proposed originally (Brass, 1975). Secondly, although it is widely assumed that the paternal orphanhood method is less robust than the maternal orphanhood method, neither the reasons why this is so nor the accuracy of the results have been investigated in detail (Brass, 1975; Hill and Trussell, 1977; Hill, 1984, Palloni and Heligman, 1985). Thirdly, Brass and Hill (1973) do not demonstrate that the relationship between the ${}_np_b$ ratios and ${}_5S_x$ proportions, which they estimate using Brass's General Standard, holds at other levels of mortality. These issues are considered in this section of the article. The findings reveal that a better approach exists to the estimation of men's mortality from orphanhood than either the weighting factors or a regression model of the form proposed for women's mortality.

The advantage of a system of weighting factors, rather than one of multipliers, is that it reduces the sensitivity of the estimates to variation in age patterns of mortality and child-bearing from the standard patterns used to derive the weights. If the rate of decrease in parental survival around age n differs from that assumed, the relationships between ${}_np_b$ and the proportions of respondents with living parents in each of the two age groups adjoining n are shifted in opposite directions. The effect of these biases on the results tends to cancel out. By evaluating equation (1) with a range of fertility and mortality models, one can establish that, at young ages (i.e., approximately when ${}_5S_x>0.5$), the impact of a different age pattern of mortality or child-bearing on the measures of interest is almost exactly proportional to the probability of *dying* between ages b and $b+n$ or its equivalent, the probability of being *orphaned* by age n. To the extent that this finding holds, such breaches in the assumptions introduce no error into the weighting factors. To express this point more formally, if the incidence of orphanhood and mortality differ from those in the standard population by a constant factor, k, so that ${}_np_b = {}_np_b^s + k(1 - {}_np_b^s)$ then the correct weight, W_n^*, equals W_n:

$$W_n^* = \frac{\{{}_np_b^s + k(1 - {}_np_b^s)\} - \{{}_5S_n^s + k(1 - {}_5S_n^s)\}}{\{{}_5S_{n-5}^s + k(1 - {}_5S_{n-5}^s)\} - \{{}_5S_n^s + k(1 - {}_5S_n^s)\}}$$

$$= \frac{(1-k){}_np_b^s - (1-k){}_5S_n^s}{(1-k){}_5S_{n-5}^s - (1-k){}_5S_n^s} = W_n \qquad (2)$$

In contrast, given these conditions, if a series of multiplying factors, m_n, is used, then the life-table estimates will all be in error by $k(m_n - 1)$.

To some extent, the derivation of regression-based approaches to the estimation of mortality from maternal orphanhood has provided justification for procedures that estimate life-table survivorship from parental survival data, controlling for the mean age at child-bearing. The goodness of fit (R^2) of models of this form is high, although the average relative error in the estimates increases with the age of the respondents and is rather large for those that are middle-aged (Palloni and Heligman, 1985).

Mammo's research has clarified the reasons why the sensitivity of the estimates increases with the age of the respondents (Mammo, 1988). He shows mathematically that, after controlling for the mean age at child-bearing, the relationship between parental survival and life-table survivorship is determined largely by the variance of ages at child-bearing multiplied by ${}_ap''_{\bar{M}}/{}_ap_{\bar{M}}$, where ${}_ap''_{\bar{M}}$ is the second differential of the probability of surviving from the mean age of child-bearing to that age plus the age of the respondents. This factor reflects the age pattern of mortality. Mammo further demonstrates that ${}_ap''_{\bar{M}}/{}_ap_{\bar{M}}$ increases with $\mu(\bar{M}+a)$ and, therefore, with a. Thus, at young ages the proportion of respondents with living parents is closely related to the parents' probability of surviving from $\bar{M}$ to $\bar{M}+a$, limiting the impact that mis-specification of either the pattern of mortality or the variance of ages at child-bearing (which is multiplied by the former factor) can have on the estimates of survivorship. As the age of the respondents increases, ${}_ap''_{\bar{M}}/{}_ap_{\bar{M}}$ becomes much larger. The mortality estimates become increasingly sensitive to errors in the assumptions about age patterns of fertility and mortality which are incorporated in the models used to derive the estimation procedure.

Mammo's findings also explain why the paternal orphanhood method is less robust than the maternal orphanhood method. The variance of the distribution of ages at child-bearing for men is larger than that for women. Moreover, partly because of the prevalence of polygyny in some parts of the world, variability in both the timing and the dispersion of male fertility distributions is far greater than that in female fertility distributions. In addition, in almost all populations, people's fathers are several years older, on average, than their mothers. Therefore, ${}_ap''_{\bar{M}}/{}_ap_{\bar{M}}$ is larger. Thus, both the likely differences between the actual characteristics of the fertility and mortality distributions and those assumed and the impact of such errors on the results are larger for men than for women. It therefore seems important to assess the sensitivity of paternal orphanhood estimates to variation in age patterns of child-bearing and mortality.

To assess the size of the biases in the mortality estimates that could arise from inappropriate assumptions, one can calculate the proportions of respondents with living fathers from known fertility and mortality schedules by evaluating equation (1) numerically. Then one can estimate mortality from these proportions using the weighting factors and compare these estimates with the schedule used to generate the parental survival data.

To determine the sensitivity of the estimates to each of the assumptions, the effects of variation in the level (α) and age pattern (β) of mortality in the relational logit model life-table system based on the General Standard (Brass, 1971), age structure (as determined by r) and the width of the fertility distribution are examined in turn, holding the other parameters constant at the values used to derive the weights. The effect of different age patterns of fertility is examined using a relational Gompertz model and a standard developed recently by "stretching" the female standard to age 80 (Paget and Timæus, 1990). The model fits male fertility distributions remarkably well. The standard fertility schedule is broadly similar in shape to the polynomial used by Brass and Hill to represent male fertility and is taken as the central pattern, although it exhibits somewhat higher fertility at late ages. A β_f of 0.675 produces a broad fertility distribution, such as those characteristic of polygynous societies, and a β_f of 1.75 a narrow fertility distribution, such as those characteristic of countries with fairly low fertility.

Table 1 shows the absolute errors in the level of mortality, as indexed by α, which result when men's mortality is estimated from the proportion of respondents with living fathers under different conditions from those assumed by Brass and Hill (1973) to derive the weighting factors. Positive errors signify that the level of mortality is overestimated and vice versa. A bias in the estimated value of α of 0.1 represents about a 1.5 year error,

TABLE 1. ERRORS IN ESTIMATES OF THE LEVEL OF MORTALITY (α) FROM PATERNAL ORPHANHOOD IN POPULATIONS WITH DIFFERENT CHARACTERISTICS, SELECTED MEAN AGES AT CHILD-BEARING ($\bar{M}$) AND CENTRAL AGES OF RESPONDENTS (n)

	$\bar{M} = 29.75$			$\bar{M} = 34.75$			$\bar{M} = 39.75$		
	$n = 10$	$n = 25$	$n = 40$	$n = 10$	$n = 25$	$n = 40$	$n = 10$	$n = 25$	$n = 40$
Mortality									
$\alpha = 0.4$	−0.016	−0.032	0.022	−0.057	−0.067	−0.039	−0.075	−0.075	0.007
−0.4	0.019	0.043	0.029	0.043	0.064	0.061	0.059	0.074	0.032
−0.8	0.036	0.096	0.101	0.078	0.132	0.152	0.111	0.160	0.105
$\beta = 0.6$	0.015	0.050	0.154	−0.011	0.037	0.180	−0.003	0.074	0.250
1.4	−0.014	−0.056	−0.139	0.009	−0.043	−0.164	−0.002	−0.077	−0.202
Growth rate									
$r = 0.5\%$	0.008	0.011	−0.004	0.013	0.012	−0.006	0.016	0.011	−0.028
3.5%	−0.010	−0.012	0.006	−0.014	−0.013	0.007	−0.016	−0.011	0.029
Fertility									
$\beta_f = 0.675$	0.079	0.052	−0.076	0.087	0.047	−0.068	0.083	0.019	−0.124
1.375	−0.040	−0.042	0.035	−0.051	−0.043	0.040	−0.055	−0.032	0.136
1.75	−0.065	−0.066	0.062	−0.078	−0.065	0.069	−0.085	−0.047	0.232
1	0.004	−0.003	−0.008	0.000	−0.006	−0.006	−0.002	−0.008	0.011

NOTE: The errors are estimated from the proportions of fathers surviving in populations in which $\alpha = 0$, $\beta = 1$, $r = 2\%$, $\alpha_f = 0$ and $\beta_f = 1$, except as explicitly varied. As the final row of the table indicates, the reference population has very similar characteristics to that used by Brass and Hill (1973) to calculate the weighting factors.

The mean ages of child-bearing given in the table are the correct ones, and the errors have been estimated using the weighting factors tabulated for $\bar{M} - 0.75$ years.

on average, in the corresponding estimate of the expectation of life at age 15.

If the actual demographic characteristics of a population differ from those assumed to calculate the weighting factors, the resulting errors in estimates of men's mortality are smaller than one might expect, given the reservations expressed about this method in the literature. As for maternal orphanhood (Palloni and Heligman, 1985), the errors increase in size with the age of the respondents, but are acceptable for the age groups used for estimation.[2] As a very approximate guide, for central ages of respondents, *n*, of 35 or less, deviations from the assumptions made in the derivation of the paternal orphanhood weights are unlikely to introduce errors into estimates of male life expectancy at age 15 of much over 1 year.[3]

Differences in the rate of natural increase of 1.5 per cent have a large impact on population age structure but an insignificant impact on the estimated level of mortality, confirming Brass and Hill's argument (1973) that the orphanhood method is very robust to errors in the assumptions made about the age distribution of the population. As the earlier discussion suggests, however, when the width of the male fertility distribution is very different from that assumed, appreciable biases are introduced into the estimates of mortality. If men's ages at child-bearing are very dispersed, the level of mortality estimated from data on young respondents will be too high and the level estimated from data on older respondents too low. Thus, the decline in mortality over time inferred from the estimates will be underestimated slightly. In contrast, if child-bearing is very concentrated, the extent of mortality decline will be exaggerated. Except among older respondents in populations with extreme mean ages at child-bearing, the errors remain fairly small.

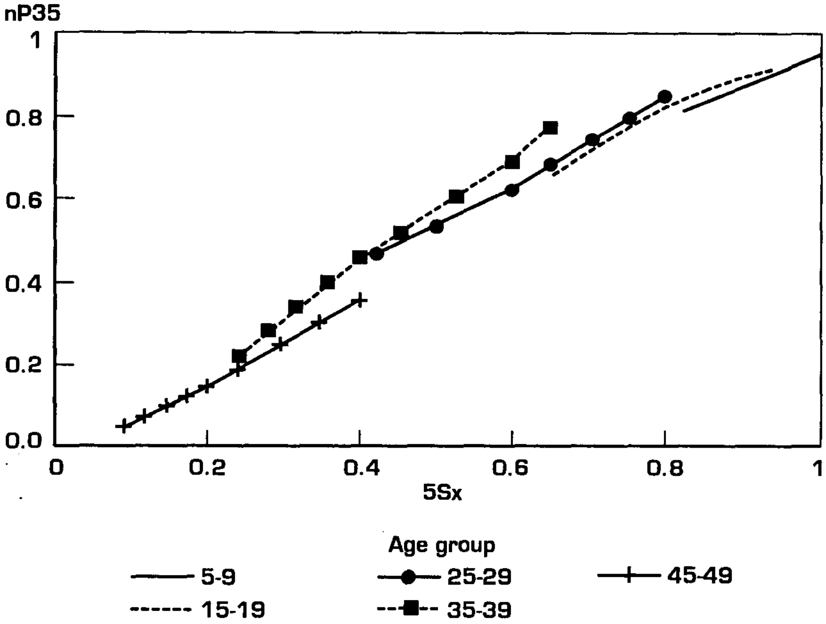
As earlier work on maternal orphanhood has suggested (Palloni and Heligman, 1985), the estimates are most sensitive to differences between the level and pattern of mortality and the standard schedule. The direction of the errors follows a systematic pattern. If mortality is higher overall than was assumed in the estimation of the weights or increases rapidly with age, the survivorship ratios estimated from parental survival are biased upwards. Mortality will be underestimated and, as the errors increase with age, the decline in mortality over time will be understated. Conversely, if mortality is lower than was assumed when deriving the weights or increases slowly with age, it will be overestimated and the decline in mortality exaggerated.

The sensitivity of orphanhood estimates obtained using the weighting factors to variation in the level and pattern of mortality is somewhat perturbing, especially as the weights represent the only method proposed hitherto for estimating men's mortality from the survival of fathers. The expectation of life at birth in the General Standard is about 43 years, which represents rather heavier mortality than is commonly found in the developing world today. In general, therefore, if reporting is accurate, the weighting factors produce slight overestimates of men's mortality. The bias is large enough to be of concern when mortality is either very heavy or very light. The fact that the size of the bias is systematically related to the level of mortality, and therefore orphanhood, suggests that it should be possible to improve the accuracy of the estimates.

The source of the bias becomes clear when life-table measures of survivorship at a range of levels of mortality are plotted against the corresponding simulated proportions of respondents with living fathers. This relationship is illustrated for several age groups in figure I using mortality schedules generated from the General Standard by varying α, which have expectations of life at birth that range from 30 to 74 years. The errors do not arise from non-linearities in the relationship between orphanhood and mortality. Except at extremely high and low mortality and in old age, the relationship is close to linear.

In fact, the bias in orphanhood estimates obtained using the weighting factors stems from the lack of an intercept term. The relationship *is* non-linear at the extremes, which largely fall outside the range of human experience. Thus, as mortality declines, the proportion of respondents with living fathers increases more slowly than a line fitted to a single mortality schedule and passing through the origin would suggest. The problem becomes more serious as the age of respondents increases because the proportion of them with living fathers varies more with the level of mortality.

Figure I. Relationship between paternal orphanhood and life-table survivorship at different levels of mortality in selected age groups ($\bar{M}$ = 35 years)



Thus, mortality is underestimated systematically, when it is high, and overestimated, when it is low.

This problem would not arise with an estimation procedure including an intercept term, which could model relationships such as those shown in figure I correctly. Further minor increases in the overall precision of the method would result from replacing the function used to represent fertility in the original derivation of the weighting factors with more sophisticated model fertility distributions.

Revised estimation procedures

New regression coefficients for the estimation of adult mortality from orphanhood are presented in tables 2 and 3. The coefficients have been calculated for maternal as well as paternal orphanhood to provide a consistent basis for estimation for the two sexes. They were estimated from simulated data created from relational logit model life-tables, based on the General Standard, and fertility distributions generated, using the relational Gompertz model, from Booth's standard (1984) for females and the Paget and Timæus' standard (1990) for males. These standards were developed for use in populations with high fertility. As in the original calculations, the age distribution of the parents is represented by a stable population with a rate of increase of 2 per cent.

TABLE 2. COEFFICIENTS FOR THE ESTIMATION OF FEMALE SURVIVORSHIP FROM THE PROPORTION OF RESPONDENTS WITH LIVING MOTHERS

n	$\beta_0(n)$	$\beta_1(n)$	$\beta_2(n)$	R^2	CV^a
10	−0.2894	0.00125	1.2559	0.997	0.0015
15	−0.1718	0.00222	1.1123	0.996	0.0031
20	−0.1513	0.00372	1.0525	0.995	0.0058
25	−0.1808	0.00586	1.0267	0.993	0.0088
30	−0.2511	0.00885	1.0219	0.992	0.0126
35	−0.3644	0.01287	1.0380	0.992	0.0172
40	−0.5181	0.01795	1.0753	0.992	0.0222
45	−0.6880	0.02342	1.1276	0.993	0.0271
50	−0.8054	0.02721	1.1678	0.992	0.0400

Estimation equation: $l(25 + n)/l(25) = \beta_0(n) + \beta_1(n)\bar{M} + \beta_2(n)_5S_{n-5}$

[a] Coefficient of variation = root mean squared error divided by the mean of ${}_nP_{25}$.

TABLE 3. COEFFICIENTS FOR THE ESTIMATION OF MALE SURVIVORSHIP FROM THE PROPORTION OF RESPONDENTS WITH LIVING FATHERS

n	$\beta_0(n)$	$\beta_1(n)$	$\beta_2(n)$	β_3	R^2	CV^a
10	−0.8251	0.00261	2.7269	−0.9953	0.978	0.0066
15	−0.4013	0.00576	1.5602	−0.3522	0.974	0.0117
20	−0.3329	0.01031	0.6656	0.3419	0.976	0.0172
25	−0.4726	0.01559	0.2161	0.7896	0.981	0.0216
30	−0.7056	0.02076	0.1997	0.9066	0.988	0.0239
35	−0.9153	0.02493	0.3484	0.8631	0.992	0.0261
40	−0.9950	0.02635	0.4269	0.8263	0.987	0.0466

Estimation equation: $l(35 + n)/l(35) = \beta_0(n) + \beta_1(n)\bar{M} + \beta_2(n)_5S_{n-5} + \beta_3(n)_5S_n$

[a] Coefficient of variation = root mean squared error divided by the mean of ${}_nP_{35}$.

The proportion of respondents with living mothers by five-year age group in each simulated population was calculated by evaluating equation (1) by summation over single years of age, y and a, taking the growth and survivorship functions in y at the midpoint of each year. For paternal orphanhood, the polynomial used by Brass and Hill to represent $f(y)$ has a fixed shape, implying that it can equally well be regarded as representing the distribution of men's ages at the conception of their live-born children and a fertility distribution. In contrast, the shapes of the $f(y)$ distributions produced by the relational Gompertz model vary with their mean. Therefore, it is preferable to retain the distributions' interpretation as representing fertility. Thus, the coefficients for paternal orphanhood were calculated by evaluating not equation (1) but a corresponding expression in which a and y retain identical definitions as for maternal orphanhood:

$$ {}_5S_x = \frac{\int_x^{x+5} e^{-ra} l(a) \int_s^w e^{-ry} l(y) l(y+a)/l(y-0.75) f(y)\, dy\, da}{\int_x^{x+5} e^{-ra} l(a) \int_s^w e^{-ry} l(y) f(y)\, dy\, da} \tag{3} $$

The parameters used to generate the model populations are shown below. Inspection of the residuals and of the bivariate relationships between the mean age at child-bearing and the proportions of surviving parents, on the one hand, and the predicted life-table measures, on the other, confirms that the relationships are linear and that the variance of the independent variables is unrelated to the dependent measures for both males and females.[4] Coefficients can be estimated reliably, therefore, from a fairly small set of data. The β parameter of the mortality models, which affects the age pattern of mortality, and the β_f parameter of the fertility models, which largely determines the variance of the distributions, were set to two and three different values, respectively, to check that this did not affect the relationships of central concern. It did not seem worthwhile, however, to introduce a large number of intermediary populations, when it was not proposed to model this variation. The resulting populations have life expectancies at birth that range from 36 to 73 years, averaging 55 years, and life expectancies at age 15 that vary between 39 and 62 years, averaging 50.6 years.

Parameters that define the simulated populations for estimation of the relationship between parental survival and life-table measures

Mortality
$\alpha = 0.2, -0.2, -0.6, -1.0$
$\beta = 0.8, 1.1$
Brass (1971) General Standard

Fertility—females
$\alpha_f = -0.5, -0.2, 0.1, 0.4$
$\beta_f = 0.7, 1.0, 1.3$
Booth (1984) standard

Fertility—males
$\alpha_f = -0.4, -0.1, 0.2, 0.6$
$\beta_f = 0.7\ (\alpha<0.6),\ 1.0,\ 1.3\ (\alpha>-0.4)$
Paget and Timæus (1990) standard

Age structure
$r = 2\%$

Although in the course of fertility decline the ages of women bearing children usually change from being distributed widely with a late mean to an earlier and narrower schedule, exceptions to this pattern exist. Therefore, the coefficients for maternal orphanhood were estimated using the full set of fertility distributions, resulting in a sample of 96 populations. The mean age at child-bearing of mothers falls between 23.5 and 31.5 years and averages 27.3 years. In contrast, the male fertility schedules assembled by Paget and Timæus (1990) suggest a strong relationship between the α_f and β_f parameters of the relational Gompertz model. Late ages at child-bearing are found mainly in polygynous societies that also have wide male fertility distributions. Therefore, no populations with early (but very wide) or late (but very narrow) fertility distributions were included in the set used to model the relationship between paternal orphanhood and men's mortality. A sample of 80 populations results, with mean ages at child-bearing that range from 29.3 to 40.3 years, with an average of 34.5 years.

The model used to predict women's mortality is the same as that proposed previously (Hill and Trussell, 1977; Palloni and Heligman, 1986; United Nations, 1983):

$$ {}_nP_{25} = \beta_0(n) + \beta_1(n)M + \beta_2(n)_5S_{n-5}. $$

New coefficients for this model are presented so that estimates can be made for ages, *n*, of 10 and 15 years and can be obtained for men and women using the same assumptions. There is no reason to suppose that the coefficients will perform any better than those published before. Indeed, it is reassuring to note that, although they are based on a different set of simulated populations, the coefficients are very similar to those published in *Manual X: Indirect Techniques for Demographic Estimation* (United Nations, 1983).

The equivalent regression equation for men sometimes produces poor estimates of mortality. Better results can be obtained, especially for populations with light mortality and late ages at child-bearing, by including information on a second age group in the model. It is argued earlier that use of weighting factors, rather than multipliers, increases the robustness of the orphanhood method to variation in the age patterns of mortality and child-bearing. The model proposed here for men combines this advantage of the weights with those of a regression-based approach.[5] While the proportion of respondents with surviving parents in two adjoining age groups is highly correlated, the additional parameter captures the effect of variation in the slope of their relationship with mortality, reducing the mean errors in the estimates for the older age groups across the set of populations used here by about 17 per cent.

The mean age at child-bearing of men in the simulated data is 34.5 years, which represents a reasonable central value for the developing countries as a whole. This suggests that measures of life-table survivorship from a base age, *b*, of 35 years should be related closely to the proportions of fathers surviving. Experimentation with different regression models confirmed that this statistic could be predicted as precisely as any other dependent variable and also indicated that the gains from fitting different models to populations with early and late mean ages of child-bearing would be modest. Thus, the model chosen to estimate men's mortality is:

$$ {}_nP_{35} = \beta_0(n) + \beta_1(n)M + \beta_2(n)_5S_{n-5} + \beta_3(n)_5S_n. $$

The relative errors in the estimated survivorship ratios, shown in tables 2 and 3, give some indication of the performance of the coefficients for paternal orphanhood, compared with the now well-established model for estimating women's mortality.[6] They are encouraging. For example, the precision of the estimates of men's mortality obtained from respondents aged 15-24 years is similar to that of the estimates of women's mortality obtained from respondents aged 30-34 years. The relative errors in the estimates increase rapidly at older ages. For this reason, no coefficients are presented for ages, *n*, of over 50 years for women's mortality or of over 40 years for men's mortality.

Robustness of the new coefficients

The size of the errors in estimates of α for men which result when mortality is estimated from further simulated populations by means of the procedures presented here is illustrated for a range of populations in table 4. While these results can be compared with those shown in table 1, they are not exactly equivalent. First, other things being equal, the regression method should yield better estimates in low-mortality populations and worse ones when mortality is high, since it was derived from populations with an average life expectancy at birth 12 years greater than that in the General Standard. Thus, except when their impact is being assessed, the mortality parameters used to generate the data are set to $\alpha = -0.4$ and $\beta = 0.95$, rather than to 0 and 1, respectively. Secondly, while the fertility model used to derive the weighting factors has a fixed shape, the shape of the distributions generated by the Gompertz model varies systematically as the timing of fertility changes, affecting the results slightly. Thirdly, the coefficients for men sometimes yield poor results for populations with unusual age patterns of fertility or mortality in which male child-bearing is also unusually early or late. They perform well, though, for mean ages at child-bearing of 31-37 years, a range that encompasses most developing country populations.

Bearing these *caveats* in mind, the estimates of men's mortality produced by the new regression method and shown in table 4 are, as expected, significantly more accurate than those yielded by the weighting

Table 4. Errors in regression-based estimates of the level of mortality (α) from paternal orphanhood, selected mean ages at child-bearing ($\bar{M}$) and central ages of respondents (n)

	$\bar{M} = 31$			$\bar{M} = 34$			$\bar{M} = 37$		
	n = 10	*n = 25*	*n = 40*	*n = 10*	*n = 25*	*n = 40*	*n = 10*	*n = 25*	*n = 40*
				Mortality					
α = 0	−0.059	−0.038	0.061	−0.031	−0.019	0.009	0.024	0.006	−0.062
−0.8	0.065	0.002	−0.055	0.010	0.005	−0.008	−0.020	0.033	0.024
−1.2	0.218	−0.000	−0.035	0.070	−0.074	0.032	−0.095	−0.090	0.088
β = 0.6	0.067	−0.016	−0.016	0.009	−0.063	−0.000	−0.031	−0.087	0.009
1.4	−0.087	−0.044	−0.091	−0.035	0.010	−0.109	0.047	0.064	−0.151
				Growth rate					
r = 0.5%	−0.008	−0.002	−0.040	0.003	0.025	−0.030	0.040	0.059	−0.037
3.5%	−0.031	−0.028	−0.046	−0.038	−0.012	−0.026	−0.019	0.020	−0.022
				Fertility					
β_f = 0.675				0.079	0.085	−0.051	0.087	0.085	−0.057
1	0.012	0.027	−0.041	−0.019	0.005	−0.027	−0.036	0.006	−0.010
1.75	−0.044	−0.054	−0.048	−0.095	−0.084	−0.007			

Note: The errors are estimated from the proportions of fathers surviving in populations in which $\alpha = -0.4$, $\beta = 0.95$ and $r = 2\%$, together with $\beta_f = 1$ for $\bar{M} = 34$ years, $\beta_f = 1.3$ for $\bar{M} = 31$ years and $\beta_f = 0.85$ for $\bar{M} = 37$ years, except as explicitly varied. The value of α_f is altered to produce the desired mean age at child-bearing.

factors at extreme levels of mortality. In particular, the errors in the estimates obtained from older respondents are much smaller. The new approach sometimes produces less accurate results than the weighting factors in populations with extreme age patterns of child-bearing, but the errors remain reasonably small.

The errors in estimates of women's mortality produced using the new regression coefficients follow a similar pattern to those in table 4 but are smaller. They are not shown here. As already reported in *Manual X* (United Nations, 1983), however, the regression-based estimates for the older age groups are significantly more accurate than those obtained from the weighting factors in populations subject to extreme levels and patterns of mortality.

When the new coefficients are applied to real data and the results are compared with those from other variants of the orphanhood method, the expected pattern of differences is found. The estimates of women's mortality are similar to those obtained using the weighting factors or other regression methods. When mortality is light, however, the majority of the estimates indicate lower mortality than those arrived at using the weighting factors. At the moderate levels of adult mortality now prevailing in many developing countries, the regression method of estimating mortality from paternal orphanhood usually yields estimates of life expectancy at age 15 that are about one half to one year higher than is indicated by estimates made using the weighting factors.

CONCLUSIONS

Concern has been expressed about the robustness of the method that exists for estimating adult men's mortality from orphanhood. This article assesses the procedure. It proposes a new method for estimating men's mortality from the survival of fathers, together with a consistent procedure for estimating women's mortality. Both theoretical considerations and analyses, using simulated data, suggest that the new method yields more accurate estimates than a system of weighting factors.

The analysis presented here indicates that there is an appreciable degree of uncertainty attached to paternal orphanhood estimates, even if the reports from which they are made are accurate. Such errors pale into insignificance compared with the uncertainty that exists about the level of adult mortality in countries where it has to be guessed on the basis of data concerning children (Blacker, Hill and Timæus, 1985). In particular, although it is less robust than the maternal orphanhood method, the paternal orphanhood method is less sensitive than the general tenor of the literature about the orphanhood method would suggest to variation in the distribution of men's ages at child-bearing.

Because of the way that the relationship between parental survival and life-table indices of survivorship changes with the level of mortality, a specification of the model used for estimation which incorporates an intercept term will eliminate a significant source of the bias that affects the system of weighting factors. This is particularly important for men and for low-mortality populations. Therefore, a regression-based procedure for estimating adult men's mortality from paternal orphanhood is developed for the first time. A set of estimation coefficients for women's mortality, based on consistent assumptions, is also presented. Estimation of the coefficients for men takes advantage of the recent development of a male standard for use with a relational Gompertz model of fertility.

Because the fathers of respondents of any age tend to be older than their mothers and subject to higher mortality and because of the greater dispersion of male ages at child-bearing, a regression model of the form proposed previously for maternal orphanhood does not prove to be a robust way of estimating mortality from paternal orphanhood. Atypical age patterns of fertility and mortality produce offsetting changes in the proportions of respondents with living parents in two adjoining age groups, relative to the equivalent proportion at the age that divides them. As a result, using data on two age groups to estimate men's mortality from paternal orphanhood yields better results than a model using data from only a single age group.

The kinds of procedure presented here represent a way of estimating adult mortality from orphanhood data quickly and conveniently, with a minimum of data. Yet, appreciable errors in the results can be expected in particular populations where age patterns of child-bearing and mortality differ from those used to derive the coefficients. Given the increasing diversity in demographic terms of developing country populations, it is doubtful that the best way to improve orphanhood estimates further is to produce different sets of estimation coefficients for use in populations with particular characteristics or to attempt to introduce additional parameters that capture, for example, the impact of differences in the variance of ages at child-bearing. Instead, estimates can be calculated readily from equation (1), using fertility and mortality schedules and age distributions that are appropriate for the population under study. The exact methodology can be tailored to the scope, historical depth and accuracy of the data available.

NOTES

[1] The author has discussed this conclusion with Professor Brass, who agrees with this interpretation of the weighting factors for paternal orphanhood presented in the 1973 paper. Note that the weighting factors in *Manual X* (United Nations, 1983) were reproduced from the 1973 paper without modification and are affected the same way.

[2] The errors sometimes change direction as the age of the respondents increases because of the way that the parameters affect the distribution of ages at child-bearing. For example, if the variance of ages at child-bearing is high, more young respondents are orphaned than expected because there are more elderly fathers than anticipated. Mortality will be overestimated. By the same token, however, the rate at which orphanhood increases with the respondents' age then declines to less than is assumed, because most older fathers have died already and more fathers are relatively young than is assumed. Beyond some crossover age, mortality will be underestimated.

[3] Further simulations, not presented here, suggest that empirically common combinations of demographic characteristics are more likely to produce offsetting errors in mortality estimates from orphanhood than errors that compound one another. The largest errors occur in countries with "developing country" mortality and "developed country" fertility or with the opposite combination of characteristics.

[4] As one would expect, given the origins of the data to which the models are fitted, outliers are not a problem.

[5] If the weighting factors are thought of as embodying two stages—first, the application of a multiplying factor, appropriate for age n, to convert the measures of orphanhood into measures of life-table survival, and secondly, the averaging of two of these measures—then:

$$\begin{aligned} {}_{n}p_{b} &= W_{n} \cdot {}_{5}S_{n-5} + (1 - W_{n})_{5}S_{n} \\ &= w_{n} \cdot f_{n} \cdot {}_{5}S_{n-5} + (1 - w_{n})f_{n} \cdot {}_{5}S_{n}. \end{aligned}$$

If an intercept term is added, one obtains:

$$\begin{aligned} {}_{n}p_{b} &= w_{n}(a_{n} + f_{n} \cdot {}_{5}S_{n-5}) + (1 - w_{n})(a_{n} + f_{n} \cdot {}_{5}S_{n}) \\ &= a_{n} + w_{n} \cdot f_{n} \cdot {}_{5}S_{n-5} + (1 - w_{n})f_{n} \cdot {}_{5}S_{n} \\ &= \beta_{0}(n) + \beta_{1}(n)_{5}S_{n-5} + \beta_{2}(n)_{5}S_{n}. \end{aligned}$$

Thus, a regression model using information on two adjoining age groups can be thought of as equivalent to the use of weighting factors with an intercept term.

[6] Note that the error statistics presented here seem relatively large (including those for the maternal orphanhood estimates compared with the error statistics presented by Palloni and Heligman, 1985), simply because a fairly high proportion of the set of populations to which the models are fitted have extreme characteristics. They are a pessimistic guide to the performance of the methods. When female mortality is estimated from these data, using Palloni and Heligman's coefficients, the errors are even greater.

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